Discussion

The sidewall effect of supercritical flow is more complicated and its influence more pronounced compared to subcritial flow cases. The disturbances of sidewall reach further because of the oblique shock waves. A narrow, "two-dimensional" wind tunnel for airfoil testing appears no longer appropriate for transonic research due to the influence of the shock. Tunnels with larger width will improve the situation a little, but there seems to be no easy solution to the problem.

At present there exist two types of physical models in the interpretation of the sidewall effect, the vortex model and the displacement model.[6] The present results strongly support the displacement model as most of the oil flow features observed can be explained satisfactorily by the model.

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Refinement of Higher-Order Laminated Plate Theories

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Introduction

To account for shear deformation in composite plates by the displacement approach, a number of improved plate theories have been proposed by various authors. Starting from a Mindlin-type theory that is different from classical plate theory only in that the normal to the midplane before deformation is assumed to remain straight, not normal, after deformation, these theories have progressed to the extent of considering cubic or higher-order variation of the in-plane displacements $\bar{U}$ and $\bar{V}$ and quadratic or higher-order variation of the transverse displacement $\bar{W}$, with respect to the transverse coordinate z, and satisfaction of zero shear condition on the lateral surfaces of the plate. However, the transverse shear and normal stress continuity at interfaces of the laminate is generally violated. The aim of the present note is to indicate how transverse shear stress continuity can also be incorporated without increasing the number of unknown variables by using a piecewise displacement distribution. This is illustrated with reference to a particular displacement model used in Ref. 1.

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Piecewise variation of $\bar{U}$ and $\bar{V}$, with respect to z represents well the behavior of the laminate wherein the stiffness varies drastically at an interface. This has been used earlier,[2,3] but such analyses have been confined to theories of the Mindlin type and consequently violate the zero shear condition on lateral surfaces. The present anaysis considers symmetric laminates subjected to antisymmetric loading about the midplane.

Formulation

A symmetric laminate of total thickness $2h$ made up of $2N$ specially orthotropic laminae is considered. The reference surface $z = 0$ is taken at the midplane, and the layers in the positive z direction are numbered serially from 1 to N starting from the layer nearest to $z = 0$. The constitutive law for the kth layer is assumed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{Bmatrix}^{(k)} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{12} & C_{22} & C_{23} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}^{(k)} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \end{Bmatrix}^{(k)} \tag{1a}$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix}^{(k)} = \begin{bmatrix} C_{44} & 0 & 0 \\ 0 & C_{55} & 0 \\ 0 & 0 & C_{66} \end{bmatrix}^{(k)} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix}^{(k)} \tag{1b}$$

The displacement model is chosen, for $z \geq 0$, as

$$\bar{U}(x,y,z) = -zW_{,x} - pu + \sum_{k=1}^{N-1} \phi_k \, [p - p(z_k)] \, H(z - z_k) \tag{2a}$$

$$\bar{V}(x,y,z) = -zW_{,y} - pv + \sum_{k=1}^{N-1} \psi_k \, [p - p(z_k)] \, H(z - z_k) \tag{2b}$$

$$\bar{W}(x,y,z) = W + qw \tag{2c}$$

$$\bar{U}, \bar{V}(x,y,-z) = -\bar{U}, \bar{V}(x,y,z) \tag{2d}$$

$$\bar{W}(x,y,-z) = \bar{W}(x,y,z) \tag{2e}$$

$$p = p(z) = z - (z^3/3h^2) \tag{2f}$$

$$q = q(z) = 1 - (z^2/h^2) \tag{2g}$$

where W, w, u, v, ϕ_k, and ψ_k are functions of x,y only, $H(z - z_k)$ is the Heaviside Unit step function, and the summation in u and v is extended over the $(N-1)$ interfaces, the kth interface being defined as that between the kth and $(k+1)$th laminae. The variation of $\bar{W}$ with z is small and hence is assumed to be the same for all layers. In the absence of terms containing ϕ_k and ψ_k, this displacement variation is the same as that used in Ref. 1.

It can be seen that this distribution satisfies displacement compatibility at interfaces and the zero shear condition on lateral surfaces of the plate. By enforcing shear stress continuity at interfaces, ϕ_k and ψ_k can be obtained as

$$\phi_k = r_k(-u + w_{,x}), \; \psi_k = s_k(-v + w_{,y}),$$

$$k = 1 \text{ to } (N-1) \tag{3a}$$

$$r_k = \left[\frac{C_{55}{}^{(k)}}{C_{55}{}^{(k+1)}} - 1 \right] \left(1 + \sum_{p=1}^{k-1} r_p \right)$$

$$k = 2 \text{ to } (N-1) \tag{3b}$$

Table 1 Comparison of normal displacement W^*

$L/2h$	z/h	W^*	W^*(EE)	W^*(SD)
50	0	1.039 112	1.039 421	1.036 135
	0.5	1.039 075	1.039 391	1.036 098
	1.0	1.038 964	1.039 928	1.035 987
12.5	0	1.551 204	1.557 080	1.508 983
	0.5	1.550 861	1.556 947	1.508 634
	1.0	1.549 834	1.555 859	1.507 588

Table 2 Comparison of ϵ_x^* and ϵ_z^*

$L/2h$	z/h	ϵ_x^*	ϵ_x^*(EE)	ϵ_x^*(SD)	ϵ_z^*	ϵ_z^*(EE)	ϵ_z^*(SD)
50	0.2	0.1976	0.1985	0.1992	−0.0598	−0.0485	−0.0598
	0.4	0.3960	0.3970	0.3991	−0.1197	−0.0970	−0.1196
	0.5	0.4957	0.4962	0.4994	−0.1496	−0.1212	−0.1495
	0.5	0.4957	0.4962	0.4994	−0.1496	−0.1511	−0.1495
	0.6	0.5982	0.5984	0.6002	−0.1795	−0.1823	−0.1794
	0.8	0.8039	0.8038	0.8033	−0.2393	−0.2454	−0.2392
	1.0	1.0109	1.0107	1.0090	−0.2991	−0.3098	−0.2990
12.5	0.2	0.1479	0.1627	0.1739	−0.0347	−0.0134	−0.0353
	0.4	0.3089	0.3259	0.3581	−0.0694	−0.0270	−0.0707
	0.5	0.3985	0.4078	0.4574	−0.0867	−0.0340	−0.0883
	0.5	0.3985	0.4078	0.4574	−0.0867	0.0586	−0.0883
	0.6	0.5306	0.5359	0.5631	−0.1041	−0.0858	−0.1060
	0.8	0.8074	0.8057	0.7991	−0.1388	−0.1510	−0.1413
	1.0	1.1051	1.1011	1.0764	−0.1735	−0.2361	−0.1766

Table 3 Comparison of σ_x^*

$L/2h$	z/h	σ_x^*	σ_x^*(EE)	σ_x^*(SD)
50	0.2	26.80	27.36	27.05
	0.4	53.73	54.73	54.20
	0.5	67.28	68.41	67.83
	0.5	1704.84	1706.45	1717.61
	0.6	2057.25	2057.92	2064.21
	0.8	2764.76	2764.05	2762.72
	1.0	3476.83	3475.59	3470.17
12.5	0.2	1.28	1.46	1.52
	0.4	2.68	2.93	3.13
	0.5	3.46	3.67	4.00
	0.5	85.74	87.85	98.45
	0.6	114.21	115.40	121.21
	0.8	173.83	173.45	172.04
	1.0	237.98	236.95	231.79

$$s_k = \left[\frac{C_{44}{}^{(k)}}{C_{44}{}^{(k+1)}} - 1\right]\left(1 + \sum_{p=1}^{k-1} s_p\right) \tag{3c}$$

$$r_1 = \left[\frac{C_{55}{}^{(1)}}{C_{55}{}^{(2)}} - 1\right], \; s_1 = \left[\frac{C_{44}{}^{(1)}}{C_{44}{}^{(2)}} - 1\right] \tag{3d}$$

Thus, the number of independent variables (W,u,v,w) remains the same as in Ref. 1. Use of the principle of minimum total potential to derive the governing equations and boundary conditions is straightforward and is omitted for the sake of brevity.

Example

To assess the accuracy of the present model, the problem of a (9 deg/90 deg)$_s$ strip with each lamina of thickness $h/2$ is solved. The plate is simply supported at $x = 0, L$, inifinitely long in the y direction, and subjected to a transverse load given by

$$(\sigma_z)_{z=\pm h} = \pm Q_0 \sin(\pi x/L)$$

The material properties are assumed as

$$E_L/E_T = 25, \qquad E_Z/E_T = 1$$

$$\frac{G_{LT}}{E_T} = \frac{G_{LZ}}{E_T} = 0.5 \quad \frac{G_{TZ}}{E_T} = 0.2$$

$$\nu_{LT} = \nu_{LZ} = \nu_{TZ} = 0.25$$

The solution is assumed in the form

$$W = A \sin(\pi x/L)$$

$$w = B \sin(\pi x/L)$$

$$u = C \cos(\pi x/L)$$

and A, B, and C are evaluated from the governing equations. The results are presented in Tables 1–3 in terms of the following parameters:

$$W^* = \frac{\bar{W}}{W_0 \sin\dfrac{\pi x}{L}}, \qquad \sigma_x^* = \frac{\sigma_x}{Q_0 \sin\dfrac{\pi x}{L}}$$

$$W_0 = \frac{24 Q_0 h}{\alpha^4\,[C_{11}{}^{(1)} + 7C_{11}{}^{(2)}]}, \quad \epsilon_x^* = \frac{\epsilon_x}{\epsilon_x^0 \sin\dfrac{\pi x}{L}}$$

$$\epsilon_z^* = \frac{\epsilon_z}{\epsilon_x^0 \sin\dfrac{\pi x}{L}}, \quad \epsilon_x^0 = \frac{24 Q_0}{\alpha^2\,[C_{11}{}^{(1)} + 7C_{11}{}^{(2)}]}, \qquad \alpha = \frac{\pi h}{L}$$

The results corresponding to the exact elasticity solution (EE) and a single displacement model for all layers [obtained by deleting terms with ϕ_k and ψ_k in Eq. (2), denoted by SD] are taken from Ref. 1 and included in Tables 1–3 for comparison.

Conclusions

It can be seen that the present model gives values of W^* closer to the exact solution than SD. The values of ϵ_x^* and σ_x^* are also closer to the exact solution for most values of z/h. However, ϵ_z^* is not improved. As has been pointed out earlier in Ref. 1, use of equilibrium equations to estimate τ_{xz}, σ_z and hence γ_{xz}, ϵ_z gives accurate results even with classical lamination theory, and hence it is felt that a higher-order theory need be judged only on the accuracy of the values of W, ϵ_x, σ_x (and ϵ_y, σ_y, γ_{xy}, τ_{xy} for the more general case of bidirectional bending).

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